# International Economics (2200-F18) 

July 5, 2018

Exam - Summer 2018-with answers.

## Problem 1

Answer whether each statement is true, false or uncertain. Defend your answer! Answers without comments can at most get half points.
1.1. Consider a Ricardian model with two sectors, one factor of production, and two countries. Let productivity in the foreign country increase uniformly across both sectors, but insufficiently to change comparative advantage. This will benefit both home and foreign.

False: It is true that a more productive foreign will benefit home because the product they import will now become cheaper. However, for foreign there are two effects: higher productivity and lower price of the good they export. The latter can dominate leavning foreign worse off.
1.2. Consider one country which trades with the rest of the world and is described by the two-factor model with capital and labor. Keep the world price fixed. Suppose there is a positive immigration inflow but that these immigrants are wealthy and bring with them more capital per person than the native population. This will decrease production of the capital-intensive good but keep the wage and return on capital constant.

False: This is the Rybczynski theorem, but due to the fact that immigrants are bringing in a lot of capital the capital / labor ratio will increase. This will increase the production of the capital-intensive good.
1.3. Consider the Dornbusch/Fisher/Samuelson model of continuous goods and two countries. Suppose there is an increase in productivity abroad. This will benefit home.

False / uncertain. Increases in productivity which are close to the "indifferent" product will lower home welfare whereas increases in productivity that are concentrated where the two countries do not compete will increase home welfare. Without more info one cannot say.
1.4. Suppose Denmark and Spain are modeled as the Heckscher-Ohlin model. Both countires are in the cone of diversification (they produce both goods). Trade is costless. Denmark is capital abundant. Then wages will be higher in Spain

False. There is factor price equalization in the Heckscher-Ohlin model.
1.5. The classical trade models (the Heckscher-Ohlin model and the Ricardian model) are suited to explain all three of the following facts: The increase in income inequality in the developed world, the decrease in the labor share in the developed world, the rise in income inequality in the deveveloping world.

False: Whereas the Heckscher-Ohlin model predicts both a rising skill-premium (and thereby inequality) as well as a decline in the labor share in the developed world it would also predict declining inequality in the developing world.
1.6. Brexit is expected to help low-wage workers

False: For two reasons. Low-wage workers do not disproportionately consume imported goods and second trade with the European Union is not best described by the HO model which would predict this. A model of increasing returns to scale and increasing variety is better suited.
1.7. Total welfare in the world is always improved when countries form regional trade agreements

False: Regional trade agreements create additional trade between the members, but it diverts trade from countries that are not joining members. The latter effect is negative and can dominate (though is not likely to do so in practice). Though, we did not show it in class total welfare of the countries that join can even decline.
1.8. Imposing an import quota or imposing and import tariff are equivalent when markets are competitive and the home government sells the quota (and gets the revenue).

True: Two reasons why import quotas might be worse is when there is market power or when the home government lets foreign governments get the revenue.
1.9. In a competitive market, a foreign country strictly prefers a voluntary export constraint to an import tariff that reduces imports by the same amount.

True: Terms of trade-terms go in the opposite direction (they improve for the foreign country under export constraints but deterioate for the tariff). This is a benefit regardless of whether the constraints are imposed by the foreign companies themselves or by the government.

## Problem 2

Consider a market in country $C$ with an inverse demand function of:

$$
p(D)
$$

where $p$ is the price that results from total consumption of $D$ units. $p^{\prime}(D)<0$ and $p^{\prime \prime}(D) \leq 0$.

Country $C$ does not itself have a firm that can service this market. Country $A$ and $B$ each have one firm that can. They each have constant marginal costs of $c_{A}$ and $c_{B}$, with $c_{A} \geq c_{B}$ where for simplicity we suppose that $c_{A}$ and $c_{B}$ are close enough that both firms will be producing throughout. The two firms choose how much to produce, $q_{A}$ and $q_{B}$, simultanously, i.e. they play a simultanous move game where actions are quantitites.
a) Show that (a) Nash equilibrium is given by:

$$
\begin{aligned}
& p^{\prime}\left(q_{A}+q_{B}\right) q_{A}+p\left(q_{A}+q_{B}\right)-c_{A}=0 \\
& p^{\prime}\left(q_{A}+q_{B}\right) q_{B}+p\left(q_{A}+q_{B}\right)-c_{B}=0
\end{aligned}
$$

and show what the necessary constraint is for firm $A$ to have positive production.
Answer: The maximization problem for firm $A$ is :

$$
\max _{q_{A}} p\left(q_{A}+q_{B}\right) q_{A}-c_{A} q_{A}
$$

where the first order condition is

$$
p^{\prime}\left(q_{A}+q_{B}\right) q_{A}+p\left(q_{A}+q_{B}\right)-c_{A}
$$

and analogously for firm $B$. To ensure that both first order conditions are binding we must ensure that firm $A$ would want to produce when firm $B$ is a monopolist (i.e. there is no equilibrium with just firm $B$ producing). That is:

$$
\begin{gathered}
p^{\prime}\left(q_{B}\right) q_{B}+p\left(q_{B}\right)-c_{B}=0 \\
p\left(q_{B}\right)>c_{A}
\end{gathered}
$$

which gives:

$$
c_{B}-p^{\prime}\left(q_{B}\right) q_{B}>c_{A}
$$

where $q_{A}$ is the production of firm $B$ if it were a monopolist.
b) Show that the response functions ( $A$ 's best response to production by firm $B$ and $B^{\prime} s$ best response to production by firm $A$ ) are downward sloping and argue that this means the equilibrium is unique. Interpret

Answer: Write the first order condition for firm $A$ as:

$$
\phi\left(q_{A} ; q_{B}, c_{A}\right)=p^{\prime}\left(q_{A}+q_{B}\right) q_{A}+p\left(q_{A}+q_{B}\right)=0
$$

which defines the quantity $q_{A}\left(q_{B}, c_{A}\right)$ as a function of $q_{B}$ and $c_{A}$. The second order condition requires $\partial \phi / \partial q_{A}<0$. Then differentiate wrt $q_{B}$ to get:

$$
\begin{gathered}
\frac{\partial \phi}{\partial q_{A}} \frac{\partial q_{A}\left(q_{B}, c_{A}\right)}{\partial q_{B}}+\frac{\partial \phi}{\partial q_{B}}=0 \Leftrightarrow \\
\frac{\partial q_{A}\left(q_{B}, c_{A}\right)}{\partial q_{B}}=\frac{\frac{\partial \phi}{\partial q_{B}}}{-\frac{\partial \phi}{\partial q_{A}}}
\end{gathered}
$$

which will have the same sign as $\partial \phi / \partial q_{B}$. This is:

$$
\frac{\partial \phi}{\partial q_{B}}=p^{\prime \prime} q_{A}+p^{\prime}<0
$$

which is negative by assumption. There are two effects on the production of firm $A$ from an increase in production by firm $B$ : Higher $B$ will reduce the price which will make production less profitable by itself. But it will also increase the negative effect on the price of higher production, i.e. $\left(p^{\prime \prime}<0\right)$.

To show that the equilibrium is unique we in addition require that one best response function is always steeper at the intersection. For this write:

$$
\frac{\partial q_{A}\left(q_{B}, c_{A}\right)}{\partial q_{B}}=\frac{\frac{\partial \phi}{\partial q_{B}}}{-\frac{\partial \phi}{\partial q_{A}}}=-\frac{p^{\prime \prime} q_{A}+p^{\prime}}{p^{\prime \prime} q_{A}+2 p^{\prime}}>-1
$$

By analogous argument we also have for the best response function of firm $B: \partial q_{B}\left(q_{A}, c_{B}\right) / \partial q_{A}>-1$. This means that in a figure with $q_{B}$ out the $\mathrm{x}-$ axis and $q_{A}$ out the y-axis the slope of $q_{A}\left(q_{B}, c_{A}\right)$ will always be flatter than $q_{B}\left(q_{A}, c_{B}\right)$ and therefore the equilibrium is unique. (This wasn't particularly clearly articulated in the question so full points will be given even if the argument for uniqueness is not complete).
c) Show that firm $B$ will be producing (weakly) more than firm $A$. Interpret

Answer: Subtract the two first order conditions:

$$
p^{\prime}\left(q_{A}+q_{B}\right)\left(q_{A}-q_{B}\right)=\left(c_{A}-c_{B}\right)
$$

from which it follows that $c_{A}-c_{B}$ implies $q_{A}<q_{B}$ because $p^{\prime}<0$. Higher production will reduce price which will reduce profits on existing production. If firm $B$ has bigger production this will hurt it disproportionately. It can only bear this if it has lower production costs.
d) Suppose that country A imposes an export subsidy of $s$ per unit exported to country $C$. Show that this will increase firm $A$ 's production and reduce firm $B$ 's production. Interpret

Answer: The first order condition is now:

$$
p^{\prime}\left(q_{A}+q_{B}\right) q_{A}+p\left(q_{A}+q_{B}\right)-c_{A}+s=0
$$

where a higher $s$ will increase production for given $q_{B}$ due to the first order condition. Specifically, we can define the optimal $q_{A}$ by writing the first order condition as

$$
\phi\left(q_{A}, s\right)=0
$$

such that:

$$
\frac{d q_{A}}{d s}=-\frac{\partial \phi}{\partial s} / \frac{\partial \phi}{\partial q_{A}}
$$

$\partial \phi / \partial q_{A}<0$ by the second order condition and $\partial \phi / \partial s$ is positive by inspection. Hence, $q_{A}$ will rise. We have already argued that this will reduce production by firm $B$.

In the following suppose the inverse demand function is given by:

$$
p(D)=A-B Q
$$

e) From now on, assume that both country $A$ and country $B$ impose subsidies of $s_{A}$ and $s_{B}$, respectively. Show that the equilibrium is now given by:

$$
\begin{aligned}
q_{B} & =\frac{A+\left(c_{A}-s_{A}\right)-2\left(c_{B}-s_{B}\right)}{3 B} \\
q_{A} & =\frac{A+\left(c_{B}-s_{B}\right)-2\left(c_{A}-s_{A}\right)}{3 B}
\end{aligned}
$$

Answer:
With this demand function we have:

$$
\begin{gathered}
p^{\prime}=-B \\
p^{\prime \prime}=0
\end{gathered}
$$

which gives first order conditions of:

$$
\begin{aligned}
& -B\left(q_{A}+q_{B}\right) q_{A}+A-B\left(q_{A}+q_{B}\right)-c_{A}+s_{A}=0 \\
& -B\left(q_{A}+q_{B}\right) q_{B}+A-B\left(q_{A}+q_{B}\right)-c_{B}+s_{B}=0
\end{aligned}
$$

which can then be written as the two expressions.
f) Consider equal costs $c_{A}=c_{B}=c$. The home governments $A$ and $B$ seek to maximize home welfare (export profits minus government subsidies). Formally, we set up the following game: first stage, the two governments simultanously set subsidies $\left(s_{A}, s_{B}\right)$, second stage: the two firms simultanously set quantities $\left(q_{A}, q_{B}\right)$. Consumption takes place and profits are earned. Show that this equilibrium is worse for both countries $A$ and $B$ than an equilibrium with no subsidies $\left(s_{A}=s_{B}=0\right)$. Interpret.

Answer:
Each government seeks to maximize the sum of profits of its firm minus the subsidies paid. For country $A$ this equals:

$$
p\left(q_{A}+q_{B}\right) q_{A}-\left(c_{A}-s_{A}\right) q_{A}-s_{A} q_{A}=p\left(q_{A}+q_{B}\right) q_{A}-c_{A} q_{A}
$$

which government A then seeks to maximize by choosing subsidy $s_{A}$ :

$$
\max _{s_{A}} p\left(q_{A}\left(s_{A}, s_{B}\right)+q_{B}\left(s_{A}, s_{B}\right)\right) q_{A}\left(s_{A}, s_{B}\right)-c_{A} q_{A}\left(s_{A}, s_{B}\right)
$$

where $q_{A}\left(s_{A}, s_{B}\right)$ and $q_{B}\left(s_{A}, s_{B}\right)$ are functions of the subsidies and are given in question $e)$. This gives:

$$
\left[p^{\prime}\left(q_{A}+q_{B}\right) q_{A}+p\left(q_{A}+q_{B}\right)-c_{A}\right] \frac{\partial q_{A}}{\partial s_{A}}+p^{\prime}\left(q_{A}+q_{B}\right) q_{A} \frac{\partial q_{B}}{\partial s_{A}}=0
$$

And then use the first order condition to get:

$$
-s_{A} \frac{\partial q_{A}}{\partial s_{A}}+p^{\prime}\left(q_{A}+q_{B}\right) q_{A} \frac{\partial q_{B}}{\partial s_{A}}=0
$$

This gives that the subsidy will be positive.
Next, due to symmetry the two countries will have the same subsidy and correspondingly the two firms will produce the same, $q_{A}=q_{B}=q$ and have the same profit. It suffices to show that at any equilibrium with $s_{A}=s_{B}=s>0$ a coordinated reduction in $s$ would raise profits. Clearly $\partial q / \partial s>0$

Home welfare is:

$$
p(2 q) q-c q
$$

where the first derivative is:

$$
\left\{2 p^{\prime}(2 q) q+p(2 q)-c\right\} \frac{\partial q}{\partial s}
$$

Use the first order condition from each firm when $c_{A}=c_{B}$ :

$$
p^{\prime}(2 q) q+p(2 q)-c+s=0
$$

which then implies:

$$
\left\{2 p^{\prime}(2 q) q+p(2 q)-c\right\} \frac{\partial q}{\partial s}=\left\{p^{\prime}(2 q) q-s\right\} \frac{\partial q}{\partial s}<0 \text { for } s>0
$$

Such that welfare is always declining in subsidies when they are positive.

